SENSITIVITY STUDIES OF THE EFFECT OF UNCERTAINTY IN THE ^{238}U(n,γ), ^{239}Pu(n,f), AND ^{239}Pu(n,γ) CROSS SECTIONS

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August 1971

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ABSTRACT

The effects of current uncertainties in the ^{238}U(n,γ), ^{239}Pu(n,f), and ^{239}Pu(n,γ) cross sections below 1 MeV, believed to be the most serious for fast reactors, were studied for a spherical model of a large LMFBR. Total variations found for a probably pessimistic assumption for ^{238}U σ(n,γ) uncertainty were about 3% in k, 0.10 in breeding ratio, 5% in sodium-void effect (out of a total effect of +2.7% k), and 20% in ^{238}U Doppler effect. For ^{239}Pu σ(n,f) and σ(n,γ) below 30 keV, disagreements among recent measurements (which are usually within error bars) correspond to a variation of about 1% in k, 0.05 in breeding ratio, 15% in sodium-void effect, and 12% in ^{238}U Doppler effect. Uncertainty in the ^{239}Pu fission cross section above 30 keV corresponds to a variation of several percent in k, about a ±5% uncertainty in the sodium-void effect, and minor changes in the other quantities. The most serious of these uncertainties are those in k and in breeding ratio. Improvement in knowledge of low-energy ^{239}Pu cross sections has significantly reduced the uncertainty in reactivity coefficients.

INTRODUCTION

It is generally recognized that nuclear-data uncertainties are the principal cause of unreliability in fast-reactor-physics calculations. The most important data uncertainties are commonly considered to be in the fission and capture cross sections of ^{239}Pu and in the capture cross section of ^{238}U. Because there have been a number of recent measurements and evaluations of these cross sections, it seemed to be of interest to assess the current uncertainty in the most important fast-reactor characteristics

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NOTE: This report is an expanded version of the paper of the same title presented at the Third Conference on Neutron Cross Sections and Technology, University of Tennessee, Knoxville, March 15–17, 1971.

associated with uncertainty in these cross sections. The reactor proper-
ties selected for study were reactivity, sodium-void effect, Doppler effect
of ^{238}U, and breeding ratio (B.R.).

NUCLEAR-DATA SELECTION

^{239}Pu Fission and Capture below 30 keV

A number of authors[1-7] have recently presented results for
^{239}Pu(n,f) and ^{239}Pu(n,γ) cross sections in a form that facilitates compar-
ison: a tabulation for common energy intervals ranging from 0.1 keV at
low energies to 5 keV at higher energies. Results in most cases now agree
within error bars; these error bars are sometimes rather large, however,
particularly for the capture-to-fission ratio, α. It seemed that simply com-
puting the reactor properties corresponding to the various reported cross-
section values would give a reasonable estimate of the uncertainty from
this source. Because of the cancellation of positive and negative effects,
the discrepancies obtained in this way are probably smaller than those that
would correspond to the uncertainty in individual measurements. The ref-
erence cross sections from which variations were made were those pre-
sented by Pitterle et al.[8] The energy-averaged values for ENDF/B-II[9] were
taken from Ref. 5.

The fission and capture cross sections were used directly as tab-
ulated in the various papers, with self-shielding factors calculated by
Kikuchi[10] applied to variations in the cross sections. These factors were
assumed to be independent of the data, which should be good enough for
the present purpose.

The base cross sections as calculated by MC^2 (Ref. 11) were ef-
fective cross sections in that flux-correction factors differing for dif-
ferent isotopes to take account of accidental resonance overlap were
used.[12,13] The flux-correction factors calculated by Kikuchi[10] were also
based on this method for a σ_p (scattering cross section per atom) consist-
ent with the reactor composition being studied.

^{239}Pu Fission and Capture above 30 keV

For ^{239}Pu fission and capture above 30 keV, the variation made from
the cross sections of Pitterle et al. was to lower fission and capture by up
to 16% between 40 keV and 1 MeV, corresponding to the difference between
the White[14] and Poenitz[15] (preliminary) ^{235}U fission cross sections used as
a standard. This was considered to be a representative uncertainty. The
effect of increasing alpha by 20% from 30 to 800 keV was also determined;
this is the uncertainty estimated by Greebler et al.[16]

^{238}U Capture

The ENDF/B Version I cross section[17] was used as a standard for ^{238}U capture. Upper and lower curves were constructed (see Fig. 1) which were intended to represent extreme limits for this cross section, based on available experiments and evaluations. Below 25 keV, the curves were calculated from the unresolved resonance parameters given in Table I. The parameters for the upper curve are those of Schmidt,[18] except that $D_{J=3/2}$ is 10.4 instead of 11.4 eV. This curve agrees well with the 1966 evaluation of Schmidt below 100 keV.
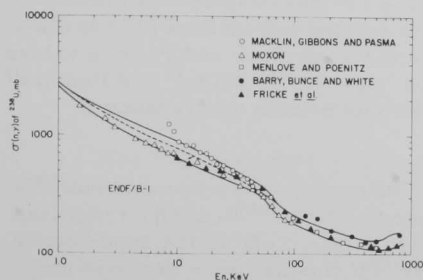
Fig. 1. Capture Cross Section of ^{238}U. The upper and lower curves are estimated extreme uncertainty limits. ANL Neg. No. 116-592 Rev. 1.

The upper curve also agrees rather well with the data of Macklin, Gibbons, and Pasma,[19] as renormalized by Davey,[20] extending up to 55 keV. Davey[20] included these measurements in his Category A of best available measurements, "Good Absolute Data." The upper curve lies from 6 to 10% above Davey's "Best Values" over most of the energy range below 100 keV. At higher energies up to 1 MeV, the upper curve agrees well with the measurements of Barry, Bunce, and White,[21] and therefore with Davey's evaluated results,[20] which were based on these measurements.

TABLE I. Unresolved Resonance Parameters Used to Calculate σ_γ of ^{238}U below 25 keV

	Upper Curve	Lower Curve	ENDF/B-I
Γ_γ, mV	24.8	24.8	24.6
$S_0 \times 10^4$	0.90	0.90	0.94
$S_1 \times 10^4$	2.5	1.0	1.58
$R \times 10^{13}$, cm	9.18	9.18	8.74
$D_{J=1/2}$	20.8	20.8	18.5
$D_{J=3/2}$	10.4	10.4	9.25

The lower curve up to 100 keV agrees well with the data of Moxon,[22] except that it is at about the lower limit of the data between 10 and 30 keV. Several recent evaluators[20,23,24] have chosen to renormalize Moxon's data upwards by factors of 1.09 to 1.15. This was justified by a desire for

consistency with certain other measurements rather than by identification of any specific deficiency in Moxon's technique, however. The shapes of both upper and lower curves below 100 keV are similar to the shape of Moxon's data. At higher energy, the lower curve agrees with the measurements of Menlove and Poenitz,[25] and above 140 keV also agrees closely with measurements of Fricke et al.[26] As pointed out by Davey,[20] the discrepancy between his evaluated values and the Menlove and Poenitz values is essentially the same as the difference between the White[14] and Poenitz[15] ^{235}U fission cross sections and probably results from discrepancy in neutron-flux monitoring.

If ratio measurements[27] among ^{235}U fission, ^{239}Pu fission, and ^{238}U capture available above 100 keV are used to obtain ^{238}U capture from the ^{239}Pu based on the White ^{235}U fission data, values close to the upper curve are obtained. If the preliminary Poenitz ^{235}U fission cross sections are used as a standard instead, values close to the lower curve are obtained up to about 600 keV. However, the error bars in the ratio measurements would allow a reduction of at least 5% from the upper curve for ^{238}U capture using the White ^{235}U fission cross section as a standard.

If the shapes of the Moxon and the Menlove and Poenitz measurements are accepted, acceptance of the Barry, Bunce, and White data requires high values of ^{238}U capture at low energy close to those of Macklin, Gibbons, and Pasma. The measurements of Fricke et al., however, which differ in shape from those of Moxon, indicate the possibility of cross sections substantially below those of Macklin et al. below 30 keV, even with agreement with the values of Barry et al. at high energies.

Davey's best values for ^{238}U capture[20] are very close to ENDF/B-I values up to 30 keV and are within plus or minus a few percent of them up to 1 MeV. The ENDF/B Version II evaluated results prepared by Pitterle[23] are about 5% below Version I over most of the range below 100 keV, the discrepancy increasing to 15% over a small range around 80 keV. Between 100 keV and 1 MeV, the two versions are in close agreement.

The evaluated values by Konshin[24] are within several percent of the Version I values up to 1 MeV. Recent estimates of the uncertainty in the ^{238}U capture cross section are (1) ±5-10% below 2 keV, ±10% between 2 and 150 keV, ±5% between 0.15 and 2 MeV;[23] and (2) ±10%.[16] The difference between the upper and lower curves used here, which were meant to represent pessimistic limits, is more like ±15% above their average over most of the range below 100 keV and ±10% up to 1 MeV. Therefore the variation in reactor characteristics obtained depending on which curve is used, should be divided by about 1.5 if consistency with these estimates of error limits is desired. The most reasonable reduction of the uncertainty assumed here is in the lowering of the upper limit between 1 and 30 keV, as there are no recent measurements to support values this high.

In calculating the effect of resonance self-shielding on the altered cross sections, only the effect on the numerator of the effective cross section was taken into account; the effect on the flux-correction factor was neglected. This causes some error in calculation of the variation in the Doppler effect, but this error is not believed to be large enough to be important for the present purpose.

METHOD OF CALCULATION

The cross-section variations were made for a spherical model of a 1000-MWe oxide-fueled fast reactor used for parametric studies of LMFBR's.[28] The 5500-liter core contained two enrichment zones of equal volume, and had 40 vol % fuel ($\rho = 0.85$), 40 vol % sodium, and 20 vol % stainless steel. The isotopic plutonium composition was 66% ^{239}Pu, 28% ^{240}Pu, 4% ^{241}Pu, and 2% ^{242}Pu. Fission products corresponding to 5 heavy at. % burnup were present, and 0.5% homogeneously distributed tantalum in the outer zone simulated shim-control effects. The core was surrounded by a 25-cm-thick blanket containing 55 vol % depleted UO_2, 30 vol % sodium, and 15 vol % stainless steel, and a 15-cm-thick reflector containing 80% stainless steel and 20% sodium. The cross sections were ENDF/B Version I, except for use of the Pitterle ^{239}Pu data and of lowered values for ^{238}U inelastic scattering.

The reference calculations were carried out with a 26-group set of cross sections with 0.5 lethargy unit width, generated by ultrafine-group MC^2 calculations for a base temperature of 1300°K and also at 2500°K for Doppler-effect calculations. Doppler- and sodium-void-effect calculations were carried out by first-order perturbation calculations, assuming a uniform temperature rise in the core in the former case and uniform core voiding in the latter.

For study of the effect of the variation of ^{239}Pu fission and capture below 30 keV, derivatives of the effect of variation of $\sigma(n,f)$ at constant α and of α at constant $\sigma(n,f)$ were obtained by direct k calculations, with additional derivatives obtained for the adjustment to critical by enrichment search. The derivatives for the energy intervals of the tabulated cross sections were obtained from those in calculation groups by linear interpolation in energy. The effects of other cross-section variations were obtained by direction enrichment search.

RESULTS OF CALCULATIONS

The importance of the variation of reactor characteristics given in Tables II and III can be judged by comparing them with what are believed to be reasonable goals for the next few years for acceptable errors from all cross-section uncertainties (see, for example, Ref. 16): reactivity, $\pm 1\%$ k; sodium-void effect, $\pm 0.3\%$ k; Doppler coefficient, $\pm 5\%$; and breeding ratio, ± 0.02.

TABLE II. Results of ^{239}Pu $\sigma(n,f)$ and α Variations

	Variation with $\sigma(n,f)$ at Constant α				Variation with α at Constant $\sigma(n,f)$				Total Variation			
	Sodium Void,[a] % k	^{238}U Doppler,[a,b] % k	Total[a] B.R.	Reactivity,[c] % k	Sodium Void, % k	^{238}U Doppler, % k	Total B.R.	Reactivity, % k	Sodium Void, % k	^{238}U Doppler, % k	Total B.R.	Reactivity, % k
Base-Pitterle $\sigma(n,f)$ and α (Ref. 8)	2.683	-0.485	1.277	100	2.683	-0.485	1.277	100	2.683	-0.485	1.277	100
					Variations below 30 keV							
ENDF/B-II (Ref. 9)	-0.112	0	-0.003	0.018	-0.092	-0.015	0.005	0.096	-0.204	-0.015	0.002	0.114
ORNL-RPI[d] (Ref. 5)												
11-g Foil	-0.004	0	0.001	-0.196	0.132	0.035	-0.031	-0.444	0.126	0.035	-0.030	-0.640
Ioniz. Chamber	-0.036	0.002	-0.001	-0.118	0.202	0.045	-0.040	-0.569	0.166	0.047	-0.041	-0.687
Saclay (Ref. 2)	-0.120	-0.001	-0.003	0.359								
Petrel[e] (Ref. 6)	0.061	0.001	0.001	0.396								
James[e] (Ref. 3)	-0.061	-0.005	0	-0.182								
Dubna[e] (Ref. 7)	-0.003	0.004	0.001	-0.665	0.185	0.028	-0.012	-0.154	+0.182	0.032	-0.011	-0.819
Schomberg et al. (Ref. 1)	-0.065	0.008	0	-0.509	-0.096	-0.015	0.012	0.193	-0.161	-0.007	0.012	-0.316
Czirr and Lindsey (Ref. 2)					0.030	0.011	-0.001	-0.067				
					Variations above 30 keV							
Poenitz $\sigma(n,f)$ (Ref. 12)	-0.256	-0.001	-0.001	-3.130								
Increase of 20% in α					-0.019	0.004	-0.022	-0.281				

[a]For enrichment search to critical, $\delta k_{Doppler}/\delta k_{Reactivity} \approx -0.0088$, where $\delta k_{Reactivity}$ is k after cross-section adjustment, $\delta k_{B.R.}/\delta k_{Reactivity} \approx +2.77$, $\delta k_{Na\ Void}/\delta k_{Reactivity} \approx 0$.
[b]Doppler temperature change 1300-2500°K.
[c]An increase of 1% k corresponds to a decrease of 1.6% in fissile inventory for an enrichment search to critical.
[d]Gaps in one set of ORNL data were filled with values from the other set. Fission data extend only to 25 keV.
[e]Data extend only to 20 keV.

TABLE III. Results of ^{238}U $\sigma(n,\gamma)$ Variations[a]

	Base Value-- ENDF/B-I $\sigma(n,\gamma)$	Upper Curve		Lower Curve	
		Above 67 keV	Above 1 keV	Above 67 keV	Above 1 keV
% δk[b]		-0.11	-0.56	1.10	2.90
Enrichment Total Fissile					
Reg. 1	0.1093	0.1095	0.1121	0.1074	0.1042
Reg. 2	0.1628	0.1631	0.1669	0.1599	0.1552
δ Sodium Void, % k	2.683	-0.015	0.112	0.107	0.035
δ ^{238}U Doppler,[c] % k					
Unmod. $\delta\sigma$	-0.485	0.001	0.052	-0.015	-0.074
Mod. $\delta\sigma$	-0.485	0.001	0.030	-0.014	-0.063
δ (B.R.)					
Core	0.896	0.001	0.029	-0.015	-0.049
Total	1.277	0.001	0.039	-0.017	-0.063

[a]Enrichment adjustment to critical.
[b]Before enrichment adjustment.
[c]Doppler temperature change 1300-2500°K.

Variations in ^{239}Pu σ_f and σ_γ

Results obtained for variations of ^{239}Pu $\sigma(n,f)$ and α are displayed in Table II. Of the results of variations in $\sigma(n,f)$ at constant α below 30 keV, only the divergence in reactivity of about ±0.5% k seems unacceptably high. This deviation is mainly due to $\sigma(n,f)$ variation above 10 keV. Perhaps the

most noteworthy result is the relatively small variation in sodium-void effect, amounting to ±0.1% k. A variation of this size is not very significant for safety, considering other uncertainties in dealing with accidents involving sodium voiding. This conclusion is in contrast with what was found from comparison calculations made in 1965, when differences amounting to ±1.0 to 1.5% k were found for total core voiding.[29] In that case, however, variations by factors as large as two occurred in choices of $\sigma(n,f)$ of ^{239}Pu in the energy region below 30 keV made by various organizations, and this is believed to be the main source of such large discrepancies in the sodium-void effect.[30] The recent data as averaged over common energy intervals agree for the most part within 10-20%.

The self-shielding factors applied to $\sigma(n,f)$ were about 0.7 at 0.1 keV, 0.8 at 0.3 keV, 0.9 at 0.7 keV, 0.93 at 1 keV, and 1.00 at 5 keV and above. No factor was applied to α, since the factors for $\sigma(n,f)$ and $\sigma(n,\gamma)$ were nearly the same.

The energy region in which the sodium-void effect is most sensitive to a given percentage change in the fission cross section at constant α is from about 100 eV up to about 2 keV, as can be seen in Table IV, in which the derivatives of reactor characteristics with respect to effective cross-section changes are given as a function of energy. Since strong fluctuations in the fission cross section occur in this energy range, significant errors in reactor calculations are possible, if the data are not properly averaged. It is desirable to have the data given in as much detail as possible as a function of energy so that the reactor physicist can perform his own averaging, taking into account the group energy structure he wishes to use, the attenuation of the neutron flux over such groups, and the perturbing effect

TABLE IV. Derivatives of Reactor Properties with Respect to Cross-section Variations per Unit Energy[a]

| Group | E, keV | ΔE, keV | $\sigma(n,f)$ at Constant α | | | | α at Constant $\sigma(n,f)$ | | | |
			% δk_{Na} $\overline{(\delta\sigma_f/\sigma_f)\Delta E}$	% $\delta k_{Doppler}$ $\overline{(\delta\sigma_f/\sigma_f)\Delta E}$	% $\delta k_{Reactivity}$ $\overline{(\delta\sigma_f/\sigma_f)\Delta E}$	$\delta(B.R.)$ $\overline{(\delta\sigma_f/\sigma_f)\Delta E}$	% δk_{Na} $\overline{\delta\alpha\Delta E}$	% $\delta k_{Doppler}$ $\overline{\delta\alpha\Delta E}$	% $\delta k_{Reactivity}$ $\overline{\delta\alpha\Delta E}$	$\delta(B.R.)$ $\overline{\delta\alpha\Delta E}$
11	40.9-67.4	26.5	0.005	0.001	0.146	-0.003	-0.001	0.0004	-0.045	-0.0027
12	24.8-40.9	16.1	0.009	0.002	0.188	-0.005	-0.002	0.0007	-0.057	-0.0035
13	15.0-24.8	9.8	-0.025	0.003	0.270	-0.007	-0.003	0.0014	-0.080	-0.0050
14	9.12-15.0	5.9	-0.021	0.005	0.392	-0.011	0.001	0.0027	-0.112	-0.0071
15	5.53-9.12	3.59	-0.021	0.006	0.468	-0.013	0.014	0.0043	-0.143	-0.0081
16	3.35-5.53	2.18	0.089	0.008	0.417	-0.013	-0.023	0.0055	-0.139	-0.0073
17	2.03-3.35	1.32	0.005	0.008	0.265	-0.010	0.005	0.0048	-0.114	-0.0048
18	1.23-2.03	0.80	-0.660	0.026	1.50	-0.063	0.294	0.0364	-0.640	-0.0314
19	0.748-1.23	0.48	-1.24	0.008	2.19	-0.083	0.582	0.0816	-0.988	-0.0434
20	0.454-0.748	0.294	-1.50	-0.034	2.24	-0.085	0.850	0.139	-1.30	-0.0380
21	0.275-0.454	0.179	-1.37	-0.073	1.84	-0.067	0.854	0.147	-1.25	-0.0380
22	0.167-0.275	0.108	-1.22	-0.127	2.04	-0.046	1.080	0.234	-1.30	-0.0305
23	0.101-0.167	0.066	-0.23	-0.091	1.06	-0.030	0.561	0.097	-0.44	-0.0167

[a]Energy in keV.

of wide scattering resonances. A representation of the fluctuations to the extent possible in terms of resonance parameters is, of course, highly desirable for accurate resonance self-shielding and overlap and ^{239}Pu Doppler-effect calculations. The latter were not attempted here because of the nonavailability of any resonance-parameter representation for the various sets of data. Experimentally, the ^{239}Pu Doppler effect appears to be small, so that this deficiency does not seem serious. The ^{239}Pu Doppler effect is defined here in the effective cross-section sense,[12,13] which is the customary definition.

The effects of a variation in the low-energy α at constant $\sigma(n,f)$ are somewhat more significant for reactivity coefficients and breeding ratio, fairly marked differences between results from the ORNL-RPI data[5] and the data of Schomberg et al.,[1] being evident. The former data are generally higher than the latter, the difference exceeding error bars in some energy regions. It was found that for variation from ENDF/B-I[31] to -II a loss in k of 0.5% occurs both for the $\sigma(n,f)$ variation at constant α and the α variation at constant $\sigma(n,f)$. The low α values of ENDF/B-I give variations of -0.16% k for the sodium-void effect and -0.03% for the Doppler effect relative to the base values. Since it is rather certain that the ENDF/B-I values are too low, they should be disregarded in establishing uncertainty limits.

Table IV indicates that the derivatives are depressed in Group 17, which contains most of the large sodium resonance at 2.85 keV. At higher energies the derivatives of reactivity coefficients with respect to cross-section changes decrease markedly, even on a per-group rather than per-unit-energy basis. Also, relative uncertainties in cross sections are smaller at high energy, so that the contribution of the region above several keV to uncertainty in reactivity coefficients is relatively unimportant. To the derivatives in Table IV there must still be added the effect of an enrichment search to critical, as given in footnote a in Table II.

Of the variations above 30 keV, the reactivity change is the most important. The corresponding decreases in k calculated for critical assemblies are unacceptably large in comparison with experiment.

Variations in ^{238}U σ_γ

The "Unmod. $\delta\sigma$" results for ^{238}U Doppler effect given in Table III correspond to neglect of the change in effective ^{238}U capture cross section with temperature. The effect of this change is smaller for the lower curve because of the weaker p-wave self-shielding with the smaller strength function. The indicated Doppler-effect variation corresponds to an uncertainty considerably less than the deviations between experiment and calculation of the order of 30% that have been observed.[32] Although other parameter

uncertainties affect Doppler-effect calculations also, the ones considered here are probably the most important. Large uncertainties in Doppler-effect calculations because of uncertainty in the cross sections considered here seem unlikely.

Use of the ENDF/B-II values would produce variations from the base values about a third of those obtained with the lower curve.

The uncertainties indicated for reactivity and breeding ratio are much too large, even after reduction to allow for an overly pessimistic choice of error limits.

ALTERNATE METHODS OF ADJUSTING FOR CROSS-SECTION CHANGES

The results given in Tables II and III are, as noted before, based on an enrichment search to critical, in which the fissile content of both core regions is changed by the same ratio. Other means of adjustment which might be more realistic in an actual case are the variation of the relative sizes of the two zones, holding the total core volume constant, and a uniform change in the core size, maintaining the ratio of the two zones constant. The final reactor properties obtained with a given initial composition and given initial zone sizes for the various ways of achieving criticality are shown in Table V.

TABLE V. Effect on Reactor Characteristics of Various Ways of Adjusting to Critical

	Region 1		Region 2				Sodium-void Effect, % δk				^{238}U Doppler Effect, % k	
Case	Enrichment Fert/Fiss Ratio	Outer Radius, cm	Enrichment Fert/Fiss Ratio	Outer Radius, cm	k	Zone	Scatt	Capture	Leakage	Total		B.R.
Initial	0.11011 8.081	86.912	0.16397 5.099	109.501	1.00454	1 2 Total	2.653 0.931 3.584	0.144 0.047 0.191	-0.210 -0.880 -1.090	2.587 0.098 2.685	-0.3861 -0.0941 -0.4808	1.2662
Enrichment search	0.10929 8.150	86.912	0.16276 5.144	109.501	1.00000	1 2 Total	2.652 0.931 3.583	0.144 0.047 0.191	-0.211 -0.880 -1.091	2.586 0.098 2.683	-0.3893 -0.0955 -0.4848	1.2765
Zone 1 radius altered	0.11011 8.081	88.181	0.16397 5.099	109.501	1.00000	1 2 Total	2.731 0.847 3.578	0.149 0.043 0.192	-0.253 -0.842 -0.1095	2.627 0.048 2.675	-0.4104 -0.0873 -0.4887	1.2771
Both zone radii altered	0.11011 8.081	85.166	0.16397 5.099	107.301	1.00002	1 2 Total	2.629 0.921 3.550	0.143 0.047 0.190	-0.224 -0.895 -1.119	2.548 0.073 2.621	-0.3844 -0.0944 -0.4788	1.2769

It is noted in Table V that there is essentially no change in any of the components of the sodium-void effect on adjusting enrichment to critical at constant core volume. There is still not much change when varying the ratio of zone volumes but holding the radius of the outer zone constant. A larger but still not very important change occurs when criticality is achieved by a uniform change in core size at constant enrichment. This

indicates that the most important effect on the scattering component is that of the variation of effective geometrical B^2 through core-size changes on the energy dependence of the adjoint function. This effect is considerably diminished when only the inner-zone radius is varied, and would be smaller still if there were more than two enrichment zones with the outer core dimension kept constant. In effect, this represents an approach to the case in which enrichment is varied by a given ratio throughout the core. There is also, of course, a change in the leakage component when core size is altered, which is considerably less when the outer core radius is kept constant. The spherical model used is not the best possible one for studying the leakage component effect, but conclusions should be qualitatively valid even in this case.

Comparison of the scattering component for the enrichment search to critical at constant core dimensions and for the search by altering both core radii shows the change in sodium-void effect when core size and enrichment are altered simultaneously. The change in scattering component with fertile-to-fissile ratio under these circumstances is comparable to that observed in earlier studies with a fundamental-mode flux with B^2 being adjusted for criticality.[13,33]

The situation is different for the Doppler effect; here the effect of varying enrichment on the amount of low-energy flux is evident, while a change in core size at constant enrichment has a smaller effect. The change in Doppler effect for a critical reactor, balancing size against core enrichment, is of the same order as observed in fundamental-mode studies.[13,34]

For the total breeding ratio, there is a significant change in adjusting the system to criticality, as this effectively amounts to a change in $\bar{\nu}$. There is little change for a critical system in balancing change in enrichment against change in core dimensions, however.

The error in critical mass corresponding to a 1% error in k is 1.9% for adjustment of relative zone size at constant total core volume, compared to 1.7% for a uniform enrichment search. If a uniform core-size change is made, the error in mass per percent k is much larger: 14%. The former type of adjustment is more likely to be made in practice than the latter, however.

CONCLUSIONS

Although a change in the method of adjusting to critical would change the results in Tables II and III slightly, the conclusions drawn from them would not be changed in any significant way. Usually, changes will be made in more than one cross section at a time with compensating effects on reactivity, so that the problem of adjustment to criticality is even less than is implied here.

Although not all possible cross-section uncertainties have been studied, it is believed that the ones considered here are the most important ones. Uncertainties in k and in breeding ratio are still far too large, with cross sections of both ^{238}U and ^{239}Pu being important contributors. Although much progress has been made in measurements of ^{239}Pu α in the last several years, the agreement among various measurements still leaves something to be desired. There has been a large reduction in uncertainty in sodium-void-effect calculations recently because of the improvement in knowledge of both $\sigma(n,f)$ and α of ^{239}Pu below 30 keV.

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